Lecture 3

6.3* - The Natural Exponential Function

We have the function $f(x) = \ln x$ which is

- one-to-one
- differentiable
- $\mathcal{D}(\ln x) = (0, \infty)$
- $\mathcal{R}(\ln x) = (-\infty, \infty)$

This means it has an inverse function, which we will call $f^{-1}(x) = \exp(x)$. Since $f(x)$ has the above properties, $f^{-1}(x)$ is

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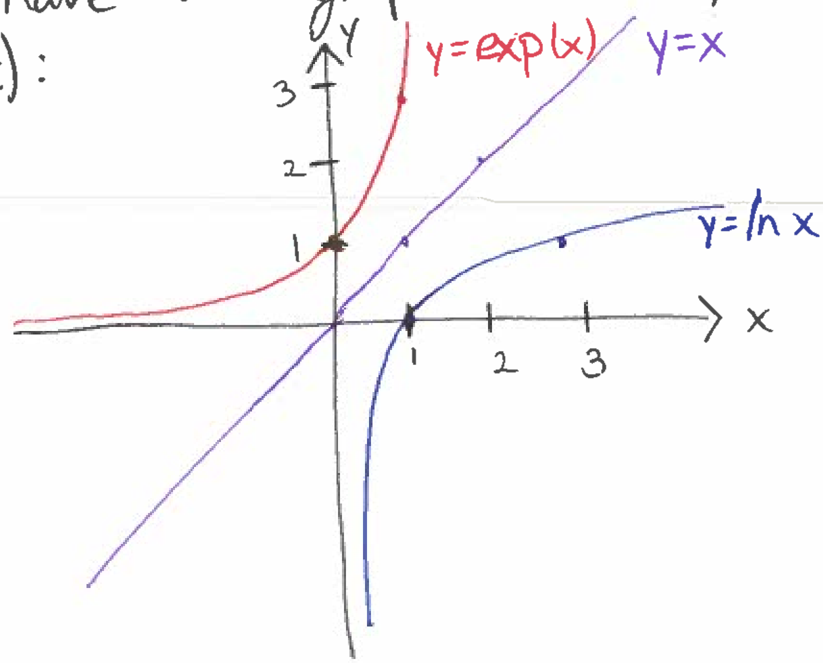
Since we have the graph of $\ln x$, we can also graph $\exp(x)$:



To get a better idea of what $\exp(x)$ is, let's look at the chart:

$y = \ln x$	$0 = \ln 1$	$1 = \ln e$	$2 = \ln e^2$	$r = \ln e^r$
$x = \exp(y)$				

This means we can write $f(x) = \exp(x)$ with a more familiar notation:

We can use that these functions are inverses to solve equations:

<u>Ex</u>: Solve for x if $\ln(2x+1) = 3$

<u>Ex</u>: Solve for x if $e^{\frac{x+1}{3}} = 7$.

Analyzing the graph of $f(x)=e^x$, we see that

$$\lim_{x\to-\infty} e^x = \qquad \& \lim_{x\to\infty} e^x =$$

Ex: Compute $\displaystyle\lim_{x\to\infty} \frac{e^x}{2e^x-1}$

From the algebraic properties of ln, we get some for e^x:

$$e^{x+y} = \qquad e^{x-y} = \qquad (e^x)^y =$$

Ex: Simplify $\displaystyle\frac{e^{x^2}e^{2x}}{(e^{x+1})^2}$

We can find the derivative of $f(x) = e^x$ using logarithmic differentiation:

And so: $\frac{d}{dx} e^x =$ & $\frac{d}{dx} e^{g(x)} =$

Also, these give us the integrals:

$$\int e^x \, dx = \qquad \& \int g'(x) e^{g(x)} \, dx =$$

Ex: Compute $\frac{d}{dx} \sin^2\left(e^{x^2+1}\right)$

Ex: Compute $\int \sec^2 x \, e^{\tan x} \, dx$